MATH 202 Differential Equations
Exam 1, Spring 2019
Duration: 70 minutes

Problem	1	2	3	4	5	6	Total
Points	16	21	28	15	15	5	100

Please circle your section:

Lecture 1 MWF 3
Moufawad

Lecture 2 MWF 11
Yamani

Lecture 3 MWF 10
Sabra

Lecture 4 MWF 2
Andrist

Lecture 5 MWF 8
Yamani

Lecture 6 MWF 1
Taghavi- Chabert

<u>INSTRUCTIONS</u>

(a) Explain your answers precisely and clearly to ensure full credit.
(b) Calculators are not allowed.
(c) Use the backside of each page if needed.

Problem 1

Let C be the unit circle $x^2 + y^2 = 1$. Find the counterclockwise circulation of the field $\mathbf{F} = e^x\mathbf{i} + (x + y^2)\mathbf{j}$ around C:

(a) (8 pts) directly (i.e. use line integral formula).

(b) (8 pts) using Green's theorem.

Problem 2

Consider the surface S with the parameterization

$$r(u,v) = u\cos v\,\mathbf{i} + u\sin v\,\mathbf{j} + v\,\mathbf{k} \qquad 0 \le u \le 1,\ 0 \le v \le 2\pi$$

(a)(7 pts) Show that the surface S is smooth.

(b)(7 pts) Find the area of S

Hint: $\int \sqrt{a^2 + x^2}\,dx = \frac{x}{2}\sqrt{a^2 + x^2} + \frac{a^2}{2} ln\left(x + \sqrt{a^2 + x^2}\right) + C$

(c)(7 pts) Integrate $\mathbf{F} = z\,\mathbf{j}$ over the surface S oriented in the direction of $\mathbf{n}$ having positive $\mathbf{k}$ component.

Problem 3

(a) (7 pts) Let C be a smooth curve in three dimensions joining the point A to the point B. Consider the field $\mathbf{F}(x,y,z) = \nabla f(x,y,z)$ with f continuously differentiable in a domain D containing C. Show that

$$\int_C \mathbf{F} \bullet \mathbf{T}\, ds = f(B) - f(A)$$

(b)(14 pts) Consider the vector field

$$\mathbf{F} = (2xe^{z}\sin y + 2y)\,\mathbf{i} + (x^{2}e^{z}\cos y + 2x)\,\mathbf{j} + (x^{2}e^{z}\sin y + 1)\,\mathbf{k}$$

Find a potential function f such that $\mathbf{F}(x,y,z) = \nabla f(x,y,z)$.

Then evaluate $\int_C \mathbf{F} \bullet \mathbf{T}\,ds$ over the curve

$$C:\ \mathbf{r}(t) = (t\cos(\pi t) - 1)\,\mathbf{i} + \pi \sin\left(\frac{\pi}{2}t\right)\mathbf{j} + (t^{2}+1)\,\mathbf{k},\quad 0 \le t \le 1.$$

(c) (7pts) Let $\mathbf{G} = (cos(xy) + ze^{x})\mathbf{i} + N(x,y,z)\mathbf{j} + (e^{x} + yz)\mathbf{k}$ be a continuously differentiable vector field.
Find $N(x,y,z)$ such that $\mathbf{G}$ is a conservative field for $y > 0$ with $N(0,y,0) = 0$.

Problem 4 (15 pts)

Find the outward flux of $\mathbf{F} = y\,\mathbf{i} + x\,\mathbf{j} + \mathbf{k}$ through the portion of the paraboloid $z = x^2 + y^2$ below the plane $z = 4$ ($\mathbf{n}$ has negative $\mathbf{k}$ -component).

Problem 5 (15 pts)

The plane $y + z = 2$ and the cylinder $x^2 + y^2 = 1$ intersect in a curve C. Suppose C is oriented counterclockwise when viewed from above. Let $\mathbf{F} = -y^2\mathbf{i} + x\,\mathbf{j} + z^2\mathbf{k}$. Use Stokes' theorem to evaluate $\oint_C \mathbf{F}.d\mathbf{r}$.

Problem 6 (5 pts)
Let $u(x,y)$ be a function defined in the plane and satisfying the following differential equation

$$u_{xx}(x,y)u_{yy}(x,y)-\left(u_{xy}(x,y)\right)^2=0$$

Assume the function u has continuous third order partial derivative.
Define the field

$$\mathbf{F}(x,y)=\left(u_x(x,y)u_{yy}(x,y)\right)\mathbf{i}-\left(u_x(x,y)u_{xy}(x,y)\right)\mathbf{j}.$$

Show that the outward flux of **F** is equal to zero across every simple, smooth closed curve C in the plane.